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Mass transfer in smooth and rough annular ducts under developing flow conditions

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Rates of solid–liquid mass transfer were measured at the inner surface of an annular duct by the electrochemical technique under developing flow conditions. Variables studied were physical properties of the solution, velocity, length of the annulus, inlet port diameter and surface roughness. Inlet port diameter was found to have no effect on the rate of mass transfer. For smooth annuli the laminar flow data fit the equation:

$$Sh = 1.029Sc^{0.33}Re^{0.55}(L/d)^{-0.472}$$

The turbulent flow data fit the equation:

$$Sh = 0.095Sc^{0.33}Re^{0.85}(L/d)^{-0.472}$$

Surface roughness in the form of V-threads normal to the flow was found to have a negligible effect on the rate of mass transfer in the laminar flow region while in the turbulent region the data fit the equation:

$$Sh = 0.167Sc^{0.33}Re(L/d)^{-0.472}(e/d)^{0.33}$$

Under the present conditions, where the dimensionless roughness height e^+ lies between 0.5 and 22, the rate of mass transfer was found to increase by an amount ranging from 10% to 200% depending on e^+.

List of symbols

a, a_1, a_2	constants
A	surface area
C	ferricyanide concentration
C_p	specific heat
d	equivalent diameter of the annulus ($d_o - d_i$)
d_o	outer diameter of the annulus
d_i	inner diameter of the annulus
d_{inlet}	inlet port diameter
D	diffusivity of ferricyanide ion
e	roughness height
e^+	dimensionless roughness height ($e\mu^*/v$)
F	Faraday's constant
f	friction factor
h	heat transfer coefficient
I	limiting current
K, K_r	mass transfer coefficient at smooth and rough surfaces, respectively.
k	thermal conductivity
L	annulus length
V	solution velocity
Z	number of electrons involved in the reaction
Nu	Nusselt number (hd/k)
Pr	Prandtl number ($C_P\mu/k$)
Re	Reynolds number ($\rho Vd/\mu$)
Sc	Schmidt number (v/D)
Sh	Sherwood number (Kd/D)
μ^*	friction velocity ($V\sqrt{f/2}$)
v	kinematic viscosity
μ	dynamic viscosity
ρ	solution density

1. Introduction

Although short concentric annular ducts are used frequently in heat and mass transfer equipment such as double tube heat exchangers, dialyzers and electrochemical reactors used to conduct diffusion controlled reactions, little work has been done on the heat and mass transfer behaviour of annuli under developing flow conditions. Previous studies have concentrated on long annuli where the flow is fully developed [1–4].

The object of the present work is to study the rate of mass transfer at the inner surface of smooth and rough annuli in the hydrodynamic entrance region. Surface roughness is one of the tools that could be used to enhance the rate of mass transfer in diffusion controlled processes. Besides, surface roughness may

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develop during equipment operation as a result of corrosion, erosion and scaling. Previous studies on the effect of surface roughness on the rate of mass transfer in tubes, ducts and annuli were carried out in the fully developed region [4–8].

Previous studies on mass transfer in smooth annuli under developing flow conditions were carried out by Singh *et al.* [9] using the dissolution of benzoic acid in water. Rai *et al.* [10] correlated the data of Singh *et al.* for the conditions: $Re > 6000$ and $L/d < 7$ by the equation:

$$Sh = 0.032[1 + (d/L)^{2/3}]Re^{0.8}Sc^{0.33}(d_o/d_i)^{0.53} \quad (1)$$

In the area of heat transfer Farman and Beckmann [11] correlated their data in the entrance region of a smooth annulus by the equation:

$$N = 0.051Pr^{0.4}Re^{0.78}(d_o/d_i)^{0.45}(L/d)^x \quad (2)$$

where $x = -1.39/Re^{0.196}$.

2. Experimental technique

The experimental apparatus (Fig. 1) consisted of a 40 litre dark Plexiglass storage tank, a plastic centrifugal pump, a vertical annular cell and the electrical circuit. The annular cell consisted of an outer stainless steel tube of 4.85 cm inside diameter and 83.5 cm length (34.4 *d*) which acted as anode, and an inner nickel plated copper cylinder of 2.45 cm diameter acting as a cathode. Anode and cathode were held in position using Plexiglass inlet and outlet sections machined as described elsewhere [4].

Three different inlet port diameters were used namely, 0.5, 1 and 2 cm. A smooth cathode and five rough cathodes were used. Roughness was created by cutting V-threads in the surface of the inner core of the annulus normal to the flow. Peak to valley heights were: 0.12, 0.26, 0.41, 0.53 and 0.72 mm. The electrical circuit consisted of a multirange ammeter and 10 V d.c power supply with a voltage regulator connected in series with the annular cell. A high impedance voltmeter was connected in parallel with the cell to measure the cell voltage. In view of the high anode area compared to the cathode area, the anode was used as a reference electrode in constructing current–voltage curves from which the limiting current was obtained.

30 litres of electrolyte were circulated between the storage tank and the cell. The solution was composed of 0.025 M potassium ferricyanide and 0.025 M potassium ferrocyanide dissolved in a large excess of sodium hydroxide. Three different sodium hydroxide concentrations were used: 1, 2 and 4 M. All solutions were prepared using distilled water and AR grade chemicals. The concentration of ferricyanide and ferrocyanide were checked by iodometry and permanganate titration respectively [12]. Before each run dissolved oxygen was removed from the solution by bubbling nitrogen gas, nitrogen bubbling was continued during experiments. Cathodes were treated as mentioned elsewhere [13].

Current–voltage curves from which the limiting current was determined were constructed by increasing the current stepwise and measuring the corresponding cell voltage. Flow rates, which ranged from 2 to 45 cm s^{-1} were regulated by means of a bypass and were measured volumetrically by means of a graduated container and a stopwatch. To study the effect of cathode height on the mass transfer coefficient, Teflon tape was used to isolate the upper part of the cathode to different lengths depending on the required cathode height. Temperature ranged from 22 to 27 °C; during each run temperature was measured and the physical properties were adjusted accordingly. Each run was repeated once or twice. Physical properties of the solution (μ, ρ) used in data correlation were taken from the data of Bourne *et al.* [14] and Berger and Hau [15] who used NaOH concentrations ranging from 0.5–3 M and 1–4 M, respectively. The fact that the present ferri-ferrocyanide concentrations differ from those used by Bourne *et al.* and Berger and Hau does not lead to a significant change in μ and ρ since these properties are determined mainly by NaOH concentration. Bourne *et al.* who used 0.01 M ferricyanide and 0.02 M ferrocyanide and NaOH concentrations ranging from 0.5 to 3 M found that their physical properties deviate by only 1.17% from solutions containing identical NaOH concentration and 0.025 M equimolar amounts of ferri and ferrocyanide. The diffusion coefficient of ferricyanide ion was calculated from the equation [14, 15]

$$D\mu/T = 2.5 \times 10^{-15} \text{kg m K}^{-1} \text{s}^{-2} \quad (3)$$

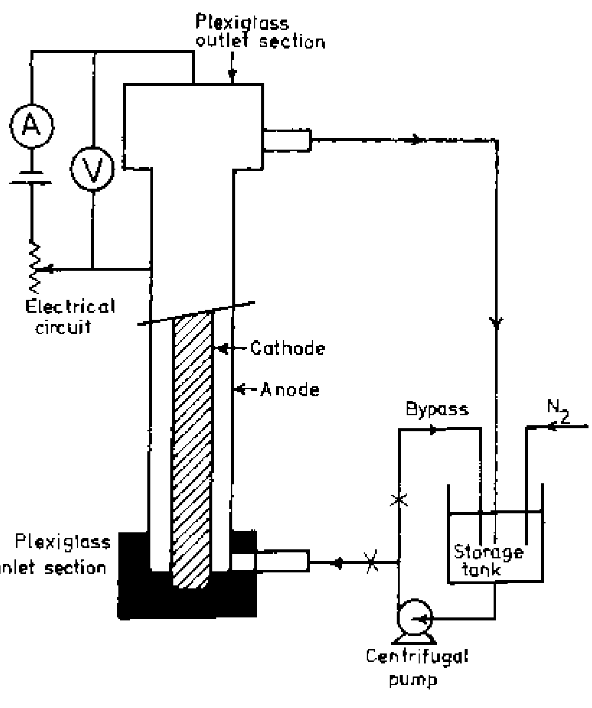

Fig. 1. Experimental apparatus.

3. Results and discussions

Limiting current was measured for different solution velocities, L/d values, inlet port diameters and degrees of surface roughness at three different values of Sc; in all cases current–voltage curves with a well defined limiting current plateau were obtained. The mass transfer coefficient was calculated from the limiting current using the equation [13]

$$K = \frac{I}{AZFC} \tag{4}$$

Figure 2 shows the effect of solution velocity on the mass transfer coefficient at a smooth annulus for different L/d.
The data fit the equations:
For laminar flow,

$$K = aV^{0.55} \tag{5}$$

for turbulent flow,

$$K = a_1 V^{0.85} \tag{6}$$

Figure 3 shows the effect of (L/d) on Sh at different Re, the data fit the equation:

$$Sh = a_2(L/d)^{-0.472} \tag{7}$$

Figure 4 shows that inlet port diameter has no effect on Sh. This is consistent with previous heat transfer studies. Farman and Beckmann [11] who studied rates of heat transfer in annular developing flow used entrance nozzle positioned on the centreline of the annulus perpendicular to the annular flow; the authors found that the entrance nozzle diameter (3/8–1 inch) has a negligible effect on the heat transfer coefficient. Sparrow and Moki [16] who studied rates of heat transfer for developing turbulent flow of air in a tube used two different axial inlet geometries of the same diameter namely, a tube inlet built into a large wall (baffle) and a free inlet framed only by the tube wall thickness. The heat transfer coefficient in the immediate neighbourhood of the inlet was higher in the case of the free inlet by a maximum of 10%.

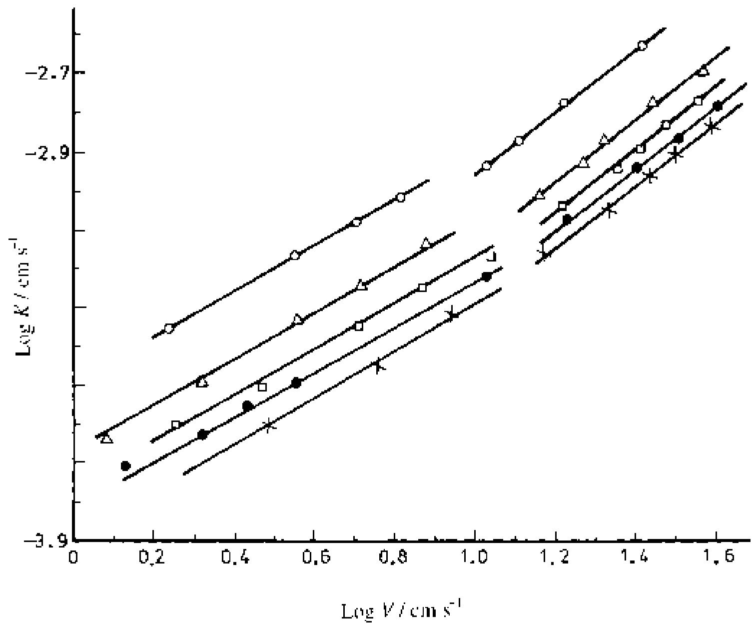

Fig. 2. Effect of solution velocity on the mass transfer coefficient at smooth surfaces. d_{inlet} = 2 cm. Sc = 1300. L/d: (○) 6.875, (△) 13.75, (□) 20.625, (●) 27.5 and (×) 34.375.

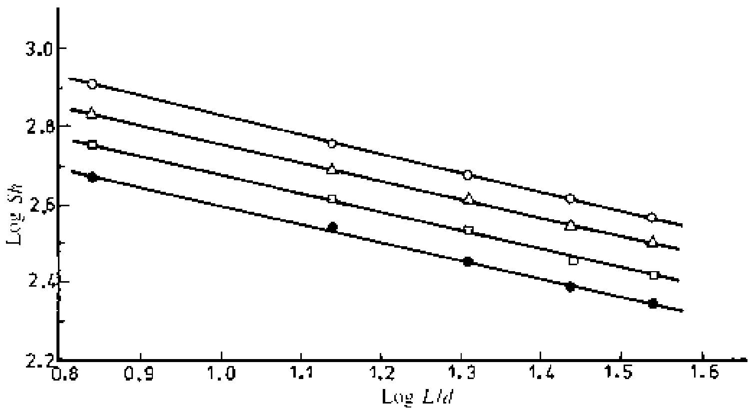

Fig. 3. Log Sh against log L/d for different Re. d_{inlet} = 2 cm. Sc = 1300. Re: (○) 8000, △ 6300, (□) 5000 and (●) 4000.

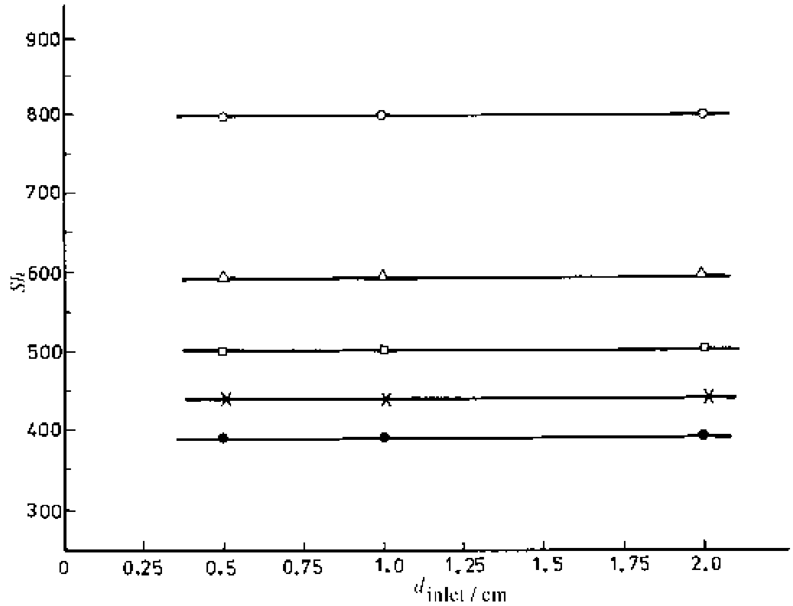

Fig. 4. Effect of inlet port diameter on *Sh*. *Sc* = 1300, *Re* = 8000. *L*/*d*: (○) 6.875, (△) 13.75, (□) 20.625, (×) 27.5 and (●) 34.375.

An overall correlation was obtained in terms of the dimensionless groups *Sh*, *Re*, *Sc* and *L*/*d*, equivalent diameter was used as a characteristic length. Figure 5 shows that the laminar flow data for the conditions 400 < *Re* < 2000; 1100 < *Sc* < 5200, 6.8 < *L*/*d* < 34.4 fit the equation:

$$Sh = 1.029Re^{0.55}\mathrm{Sc}^{0.33}(L/d)^{-0.472} \tag{8}$$

with an average deviation of ±2.6%. Theoretical analysis of heat and mass transfer in channels under developing flow [17] leads to a *Re* exponent of 0.5 and (*L*/*d*) exponent of –0.5 which agree fairly well with Equation 8.

Recently Ould-Rouis *et al.* [18] made a numerical and experimental study of the local rate of mass transfer in the entrance region of an annulus under

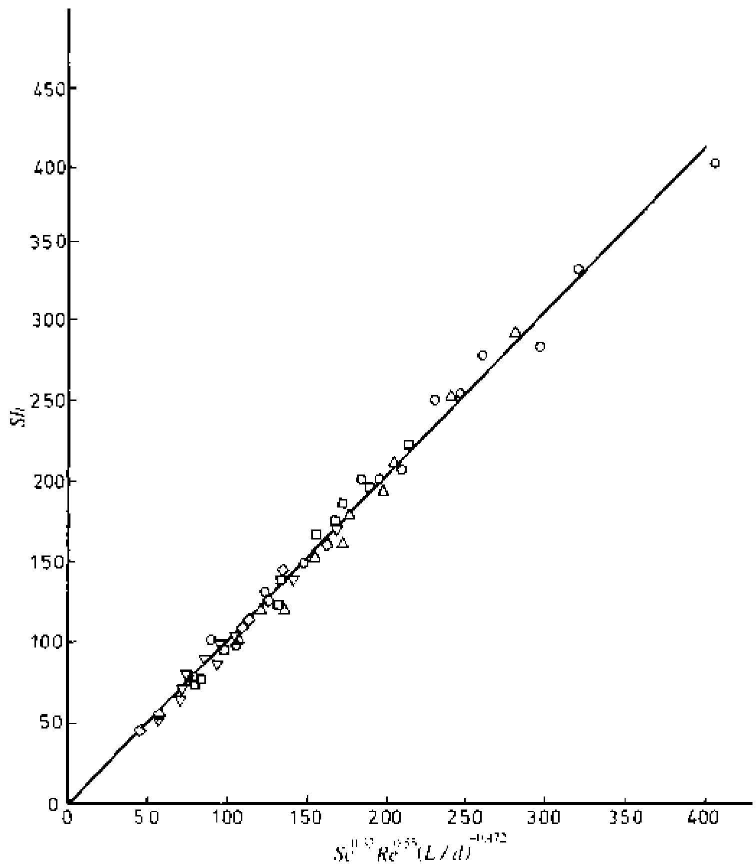

Fig. 5. Overall mass transfer correlation at smooth surface under laminar flow conditions. *L*/*d*: (○) 6.875, (△) 13.75, (□) 20.625, (▽) 27.5 and (◇) 34.375.

laminar flow using microelectrodes; the results obtained indicate that $Sh/Sc^{0.33}$ depends on Re and (d/L) raised to an exponent of 0.52, which agrees fairly well with the present results. Equation 8 compares with the equation obtained by Qi and Savinell for developing flow in parallel plate electrolytic cell under laminar flow [19]

$$Sh = 0.57Re^{0.5}Sc^{0.33}(d/L)^{0.43} \tag{9}$$

The turbulent flow data for the conditions $3000 < Re < 10\,000$; $1100 < Sc < 5200$ and $6.8 < L/d < 34.4$ fit the Equation (Fig. 6)

$$Sh = 0.095Sc^{0.33}Re^{0.85}(L/d)^{-0.472} \tag{10}$$

with an average deviation of ±2.8%. Figure 7 shows that the present turbulent flow data (Equation 10) are in good agreement with the corresponding previous heat (Equation 2) and mass transfer (Equation 1) data. Figure 8 shows the effect of solution velocity on the mass transfer coefficient at rough surfaces, the mass transfer coefficient was calculated using the geometrical area of the annulus. At low solution velocities corresponding to laminar flow the mass transfer coefficient is insensitive to surface roughness. This agrees with the finding of other investigators who studied heat and mass transfer at rough surfaces under fully developed flow. Figure 8 shows that at high velocities corresponding to turbulent flow ($Re > 2300$) the mass transfer coefficient increases with increasing e/d except for $e/d = 0.005$ which gives mass transfer coefficients close to those given by the smooth surface. The ineffectiveness of surface roughness with $e/d = 0.005$ may be attributed to the fact that

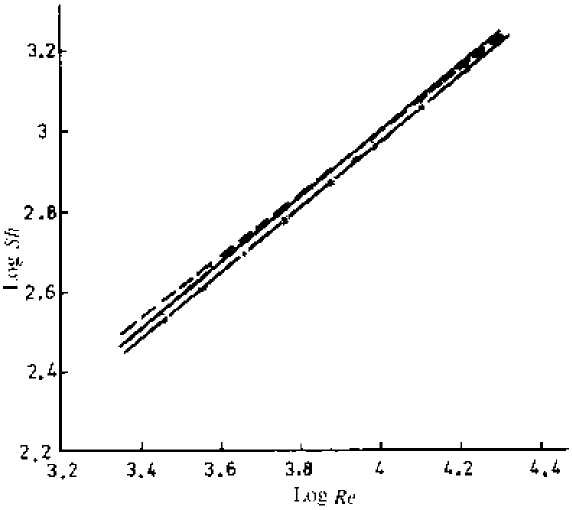

Fig. 7. Comparison of the present data with previous heat and mass transfer studies under turbulent flow conditions. Key: (-·-) present study, (- - -) Rai *et al.* [10], (——) Farman and Beckman [11].

roughness elements are submerged in the laminar sublayer of the hydrodynamic boundary layer. It seems that surfaces with $e/d > 0.005$ have peak to valley height higher than the thickness of the laminar sublayer for the present range of Re. Under such condition the diffusion layer and the laminar sublayer follows approximately the pattern of surface roughness [20], this leads to an enhanced rate of mass transfer through: (i) increase of the diffusional area, for the present roughness the increase in area over the smooth surface ranges from 35% to 40%; (ii) boundary layer separation behind surface protrusions generates eddies which penetrate the laminar sublayer

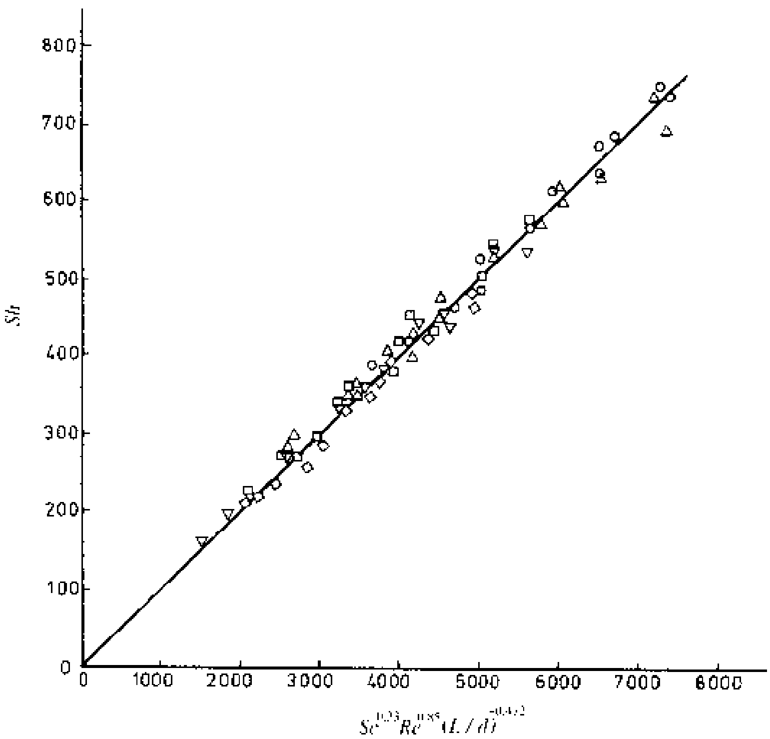

Fig. 6. Overall mass transfer correlation at smooth surface under turbulent flow conditions. K/d: (○) 6.875, (△) 13.75, (□) 20.625, (▽) 27.5 and (◇) 34.375.

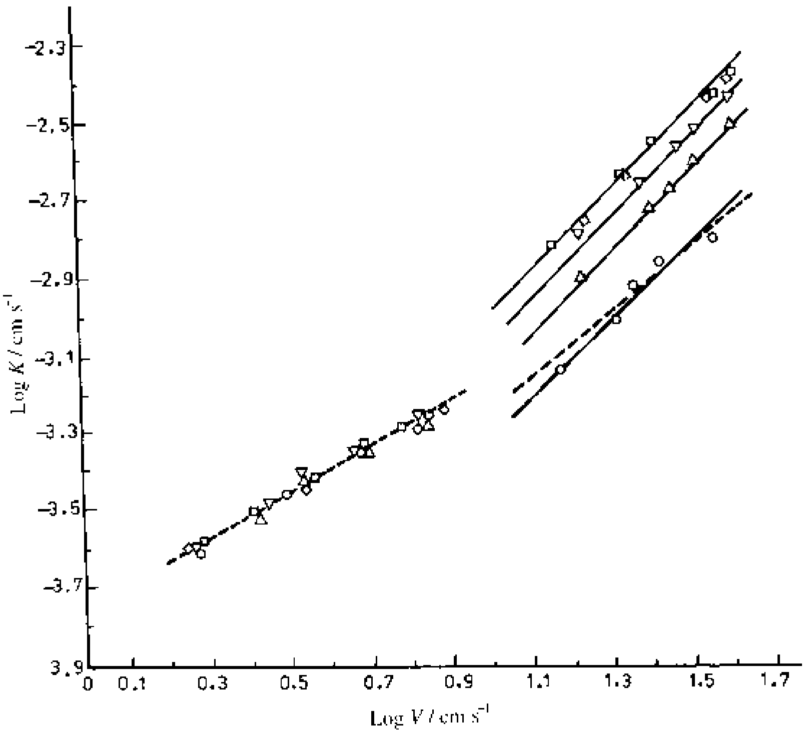

Fig. 8. Effect of solution velocity on the mass transfer coefficient at rough surfaces. Sc = 1300. e/d: (- - -) smooth, (○) 0.005, (△) 0.011, (▽) 0.017, (□) 0.022 and (◇) 0.03.

and the diffusion layer. To substantiate the above argument the ratio K_r/K was plotted against a dimensionless roughness height e^+ as shown in Fig. 9. The friction factor used in calculating e^+ was obtained for each degree of roughness at different Re from a Moody diagram [21]. Figure 9 shows that under the present conditions, surface roughness substantially increases the rate of mass transfer when e^+ exceeds 3, that is, when the roughness height becomes larger than the laminar sublayer whose dimensionless thickness y^+ extends from 0 to 3 [6]. Figure 9 shows that the present roughness elements, which increase the rate of mass transfer, lie totally in the transition region of the hydrodynamic boundary layer which extends from $y^+ = 3$ to $y^+ = 25$; that is, the fully rough region ($e^+ > 25$) has not been reached under the present conditions. Figure 9 shows that under the present conditions the maximum enhancement in the rate of mass transfer (K_r/K) reaches 2.1.

The present mass transfer data at rough annuli for the conditions $3000 < Re < 1000$; $1100 < Sc < 520$; 6.8

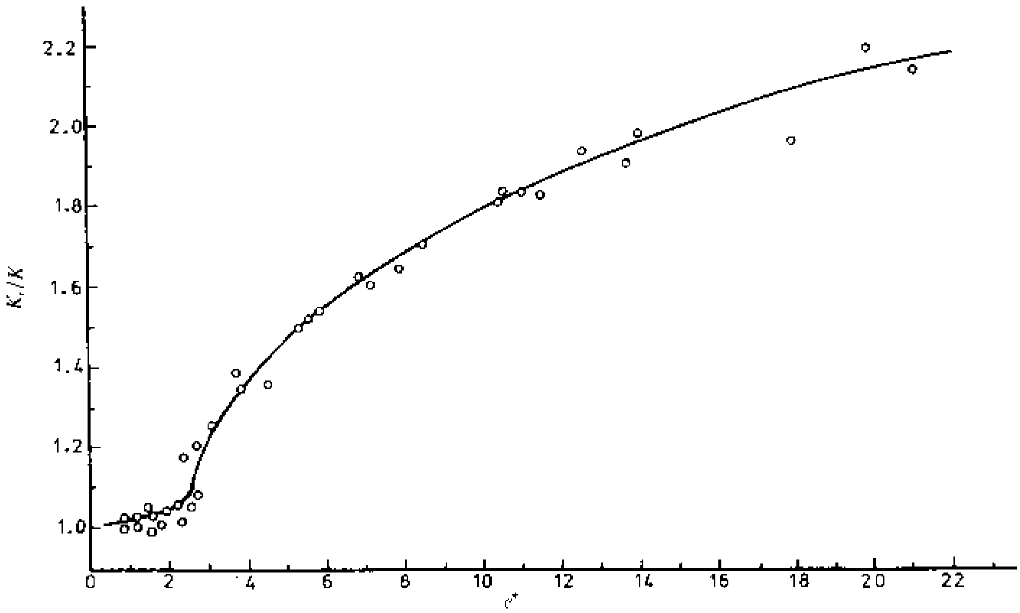

Fig. 9. K_r/K against e^+. Sc = 1300

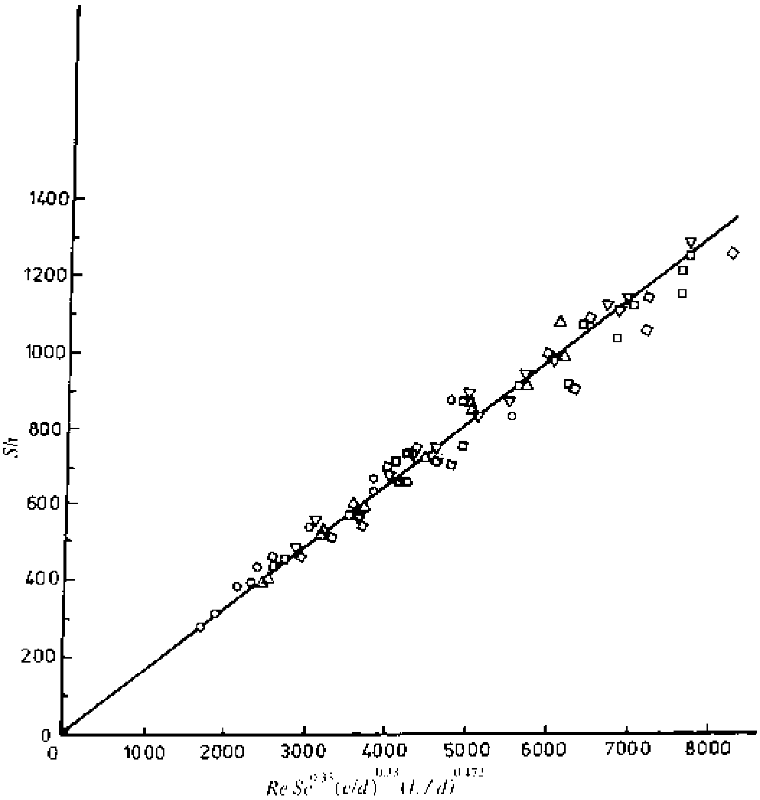

Fig. 10. Overall mass transfer correlation at rough surfaces under turbulent flow conditions. *e*/*d*: (○) 0.005, (△) 0.011, (▽) 0.017, (□) 0.022 and (◇) 0.03.

$< L/d_e <$ 34.4; and 0.005 $< e/d <$ 0.03 were found to fit the equation (Fig. 10):

$$Sh = 0.167Sc^{0.33}Re(L/d)^{-0.472}(e/d)^{0.33} \qquad (11)$$

with an average deviation of ±4.7%.

Dividing Equation 10 by Equation 11, the enhancement factor K_r/K for the present range of conditions can be expressed by

$$\frac{K_r}{K} = 1.758Re^{0.15}(e/d)^{0.33} \qquad (12)$$

The increase in the *Re* exponent from 0.85 (Equation 10) to 1 (Equation 11) due to surface roughness agrees with previous studies on the effect of surface roughness on the rate of mass transfer in tubes and annuli under fully developed flow [4–8].

4. References

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